Absolute Frequency Measurement of the Sr$^+$ Ion Optical Clock With a Fourfold Uncertainty Reduction

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***Summary*—We have made an absolute frequency measurement of the electric quadrupole clock transition of the ^{88}Sr$^+$ ion in June 2017, using a satellite link to the SI second. The measurement uptime was 92 h. The uncertainty of the measurement has been reduced by a factor of four compared to our previous best measurement that had an uptime of 100 h. The lower uncertainty of the measurement reported here is the result of an improved evaluation of the link uncertainties and of a better use of the high stability of the maser flywheel oscillator. The favorable maser noise properties allows the extension of the maser evaluation period over the full 30 days of the Circular T report of June 2017 without significant degradation of the dead time uncertainty between the maser and the optical clock. Avenues to improve future absolute frequency measurements are also discussed.**

Keywords*—*ion optical clock; absolute frequency measurement; satellite frequency transfer

I. Introduction

In the context of the international effort towards a new definition of the SI second using an optical frequency standard, it is essential to make high accuracy absolute frequency measurements of optical standards, ideally approaching the uncertainty limit set by the primary frequency standards (PFS) realized with cesium fountain clocks. These measurements will ensure that there is no discontinuity in the value of the SI second at the time of the redefinition [1].

Here we report on a preliminary analysis of the absolute frequency measurement of the $5s\,^2S_{1/2}{-}4d\,^2D_{5/2}$ clock transition of ^{88}Sr$^+$ using a GPS satellite link to the SI second.

II. Link to the SI second

Fig. 1 illustrates the frequency transfer path from the ^{88}Sr$^+$ ion clock to terrestrial time (TT), which has for unit the SI second on the geoid. The time difference between the free-running flywheel maser oscillator and UTC(NRC) is continuously measured using a time interval counter. UTC(NRC) is generated from a Cs beam clock using a microstepper and is compared to UTC / TAI using a GPS link analyzed at the BIPM using the precise point positioning (PPP) method. The deviations UTC−UTC(NRC) over five-day intervals are published in the monthly BIPM Circular T reports and are used here for the frequency determination of ^{88}Sr$^+$. TAI is calibrated using primary and secondary frequency standards (PSFS) that report to BIPM.

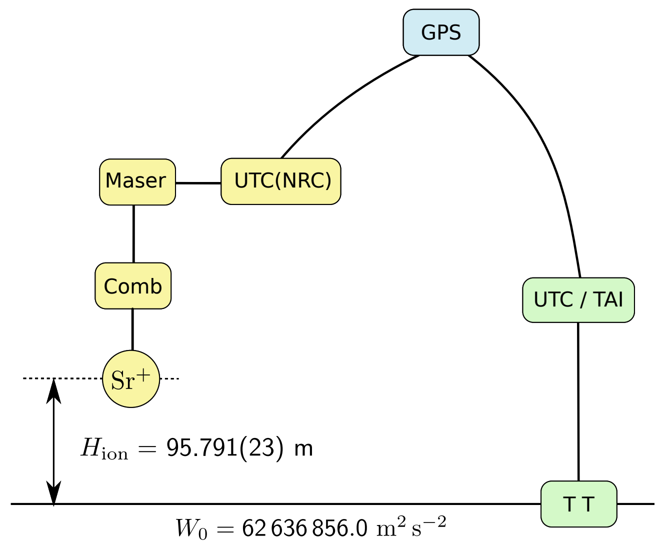

Fig. 1. Diagram illustrating the frequency transfer links between the ^{88}Sr$^+$ ion clock and TT. $H_{\rm ion}$ is the optical clock height above the geoid where the gravitational potential has the value W_0 [2].

The optical frequency was compared to the local maser using a femto-second laser frequency comb [3]. The measurements spanned approximately 12 days, from MJD 57916 to MJD 57928, where MJD is the modified Julian date. The measurement uptime was 92 h (3.82 d), limited by the intermittent operations of the optical clock and the frequency comb.

III. Uncertainty Evaluation

Table I summarizes the uncertainty contributions to the absolute frequency measurement. The ^{88}Sr$^+$ optical clock uncertainty has been discussed in detail previously [4]. The uncertainty of the optical clock reported in Table I includes both systematic and statistical uncertainties, and some other minor corrections that account for variations in the experimental conditions compared to the previous study.

TABLE I

SIMPLIFIED UNCERTAINTY BUDGET OF THE $^{88}Sr^+$ ABSOLUTE FREQUENCY MEASUREMENT FOR TWO DIFFERENT MASER EVALUATION PERIODS. UNCERTAINTIES ARE GIVEN IN UNITS OF 10^{-16} FRACTIONAL FREQUENCY.

Source	**Uncertainty**	
	15 d	30 d
$^{88}Sr^+$ optical clock	0.114	0.114
Gravitational red shift	0.026	0.026
$^{88}Sr^+$ − Maser (dead time)	1.60	2.42
UTC(NRC) − TAI	4.87	2.61
TAI − TT	4.08	2.20
Total	6.55	4.19

The gravitational red shift of the optical clock must also be considered since the SI second is defined on the geoid. The $^{88}Sr^+$ clock is located at an orthometric height of $H_{\mathrm{ion}} = 95.791(23)$ m above the geoid, causing a fractional frequency shift of $104.515(26) \times 10^{-16}$ when considering the local gravitational acceleration [4].

The other uncertainties listed in Table I are related to the frequency links and depend on the maser evaluation period. If one chooses a 15 d period (MJD 57914 – 57929) that closely matches the measurement campaign, the dead time uncertainty between the optical clock and the maser is kept to a minimum, here 1.60×10^{-16}. However, the uncertainties of the GPS link and of the evaluation of TAI increase. The total uncertainty obtained in this case is 6.55×10^{-16}. On the other hand, an evaluation period of 30 d (Circular T 354, MJD 57904 – 57934) increases the ion-maser dead time uncertainty to 2.42×10^{-16}, while the other link uncertainties are reduced. The longer evaluation period gives a significantly lower uncertainty of 4.19×10^{-16}. The optimum choice of evaluation period is determined by the maser noise characteristics and the measurement uptime pattern.

IV. RESULTS

The new determination of the *S–D* frequency of $^{88}Sr^+$ is compared with previous values in Fig. 2. The uncertainty evaluation of 4.19×10^{-16} is four times lower than our previous determination [4] labeled NRC 2016 in the figure. It also has a slightly lower uncertainty than a measurement made using a local cesium fountain clock (NPL 2014), where the reported uncertainty was 5.3×10^{-16} [6].

Fig. 2 shows the excellent agreement between the frequency reported in this work and our previous measurements, but there is a clear disagreement with the recommended value [5].

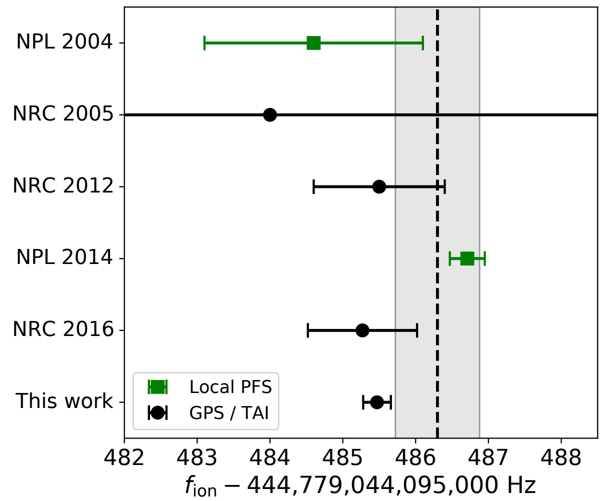

Fig. 2. Comparison of absolute frequency measurements of the $5s\,^2S_{1/2}$–$4d\,^2D_{5/2}$ transition of $^{88}Sr^+$ made with either a local cesium fountain clock or with a satellite link to the SI second. The 2021 CIPM recommended frequency is shown as the vertical dashed line. The uncertainty of the recommended value is represented by the light gray area [5].

V. CONCLUSIONS

A preliminary new determination of the absolute frequency of the $^{88}Sr^+$ ion clock was made using a GPS link to the SI second. An improved evaluation of the dead time uncertainty between optical clock and maser combined with an extended evaluation period (30 d) of the maser have resulted in a fourfold decrease in measurement uncertainty, from 17×10^{-16} [4] to 4.19×10^{-16}. The extension to 30 d despite a 12 d measurement period was possible because of the low noise characteristics of the maser.

Table I shows that three contributions dominate the uncertainty budget. The dead time uncertainty can be reduced significantly with a measurement campaign that spans a full Circular T evaluation period instead of the 12 days used here. The uncertainty of TAI is 1.1×10^{-16} for November 2021, a factor of two lower than during the June 2017 measurement campaign. This is a consequence of the increased number of PSFS's that contribute to TAI. It would therefore be quite feasible to measure the $^{88}Sr^+$ frequency with an uncertainty of 2.5×10^{-16} with a one month measurement campaign. Lower uncertainties could be achieved with an integer ambiguity resolution PPP (IPPP) analysis [7] and with a measurement campaign that spans more than one month.

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